Tunneling between two-dimensional electron systems in a high magnetic field: role of interlayer interactions

F. D. Klironomos and Alan T. Dorsey
Department of Physics, University of Florida, P.O. Box 118440, Gainesville, Florida 32611-8440
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We calculate the tunneling current for a bilayer quantum Hall system in the interlayer incoherent regime. In order to capture the strong correlation effects we model the layers as two Wigner crystals coupled through interlayer Coulomb interactions, treated in the continuum limit. By generalizing previous work by Johansson and Kinaret (JK), we are able to study the effect of the low energy out-of-phase magnetophonon modes on the electron "shake-up" which occurs during a tunneling event. We find the tunneling current peak value to scale with the magnetic field as found by JK; however, we find a different scaling of the peak value with the interlayer separation, which agrees with the measurements by J. P. Eisenstein *et al.*, *Phys. Rev. Lett.* **74**, 1419 (1995).

I. INTRODUCTION

As a consequence of interlayer interactions, bilayer two dimensional electron systems can exhibit novel behaviors in the high magnetic field (quantum Hall) regime. The virtue of the bilayer system is that the relative strength of the intralayer to the interlayer interactions can be varied; their relative strengths are characterized by the dimensionless quantity d/l, with d the interlayer separation and l the magnetic length. For example, a balanced bilayer system (*i.e*, with equal densities) at total filling factor $\nu_T = 1/2$ (each layer in a $\nu = 1/4$ state) or $\nu_T = 1$ (each layer in a $\nu = 1/2$ state) exhibits quantum Hall plateaus which would have been forbidden for the single layer system.[1,2] A spectacular manifestation of interlayer interactions for $\nu_T = 1$ is the onset of spontaneous interlayer coherence, which occurs for closely spaced layers (with $d/l < 1.8$). The interlayer coherent regime is characterized by a prominent zero bias peak in the tunneling conductance,[3] which disperses with the application of an in-plane magnetic field.[4] Recent transport studies provide compelling evidence that the coherent state can be understood as a bilayer excitonic condensate.[5,6]

Here we will focus instead on the tunneling properties in the *incoherent* regime. Even in the absence of interlayer coherence there are still important effects of the interlayer interactions, in particular the Coulomb barrier peak[3,7] which is observed in the tunneling current-voltage characteristic (I–V). This behavior is a consequence of both the interactions and the electron dynamics in high magnetic fields. A semiclassical picture is that the electrons in the accepting layer accommodate a tunneling electron by forming a correlation hole. However, the cyclotron motion of the electrons in a high magnetic field frustrates the formation of this hole (rather than moving radially outward to form the hole the electrons in the accepting layer precess, due to the Lorentz force), and the tunneling can only proceed if the tunneling electrons are externally provided with sufficient energy to overcome this "Coulomb barrier". The result is a peak in the tunneling current at a bias equal to the typical Coulomb energy, such that $eV \approx e/4\pi\epsilon a_0$, where a_0 is the average spacing between electrons. The width and shape of the I–V characteristic will depend upon the interplay of intralayer versus interlayer interactions (*i.e.* on the parameter d/l), and is the subject of this paper. The semiclassical picture described above is essentially a dynamical version of a static exciton picture of the tunneling process, first discussed by Eisenstein *et al.*,[8], in which the tunneling electron and the hole that it leaves behind combine to form an exciton; hints of the exciton's existence emerge in shifts of the I–V characteristics with interlayer spacing (the Coulomb barrier peak is "red-shifted" to a lower potential by virtue of the interlayer Coulomb energy $-e^2/\epsilon d$).

To study the tunneling current we adopt a model proposed by Johansson and Kinaret[9] (JK), in which the two dimensional electron system in each layer is assumed to have formed a Wigner crystal (WC). Of course, for the experimentally realized filling factors the layers are in a liquid phase; the JK model is only intended to capture the important short distance correlations among the electrons, and the magnetophonon modes of the WC provide a reservoir of low energy excitations that can dissipate the energy of the tunneling electron. In the approach of JK the two layers were assumed to be noninteracting; we depart from their approach by incorporating the neglected interlayer interactions. These interactions *qualitatively* change the collective mode structure of the bilayer system by introducing a gap in the out of phase magnetophonon mode and by affecting dramatically the I–V characteristics of the bilayer system since, as we mentioned earlier, they "red-shift" the peak bias values by an amount proportional to the excitonic energy $e^2/\epsilon d$. The tunneling electron couples to the out-of-phase magnetophonon mode, and the electronic "shake-up" which accompanies the tunneling event depends upon the behavior of this mode.

This paper is organized as follows. In Sec. II we review the continuum theory of the single layer WC. In Sec. III we generalize this to the bilayer case, including both long range and short range correlations between the two layers, and find the collective modes for the bilayer WC. We also determine the coupling between a tunnel-

ing electron and the collective modes of the bilayer WC, which can be written as a type of spin-boson model.[10] Section IV is devoted to the calculation of the tunneling current, which is expressed in terms of a correlation function that is determined by solving a linear integral equation both numerically and with an approximate analytical method. The solution allows us to qualitatively reproduce the tunneling current curves and obtain a good theoretical estimate for the peak bias values. It also reproduces the expected peak bias voltage dependence of the I–V curve with interlayer distance. Many of the details of our calculations are provided in the appendices.

II. SINGLE LAYER THEORY

We will start by introducing the single layer model, in which the 2-D electron system is assumed to be in a Wigner crystal state, modeled as an elastic medium with a momentum cut-off of $q_0 = 2\sqrt{\pi n_0}$, where n_0 is the single layer density (the cut-off is determined by imposing conservation of the total number of states). Of course, for the filling factors relevant to the tunneling experiments this is not the ground state of the true system. This simplified approach to the highly correlated liquid–like state of the electron system takes care of the short range correlations among electrons and creates an abundance of low energy collective excitations, which are needed to relax the local defects created by tunneling events. In order to describe the system as an elastic medium we need to introduce the two Lamé coefficients (the compressional and shear moduli) λ, μ which are associated with the energy cost of deformations in the system.[11] The effect of the perpendicular magnetic field is introduced through the vector potential $\mathbf{A}$. The Lagrangian that describes the dynamics of the 2-D electron system in the presence of the perpendicular magnetic field and the intralayer Coulomb interaction is given by

$$L = n_0 \int d^2r \left\{ \frac{1}{2} m \dot{\mathbf{u}}^2 - e\dot{\mathbf{u}} \cdot \mathbf{A}(\mathbf{u}) - \frac{\lambda}{2n_0}(\partial_i u_i)^2 - \frac{\mu}{4n_0}(\partial_m u_l + \partial_l u_m)^2 + \frac{1}{2} n_0 \int d^2r' \frac{e^2 [\nabla \cdot \mathbf{u}(\mathbf{r})][\nabla' \cdot \mathbf{u}(\mathbf{r}')]}{4\pi\epsilon |\mathbf{r} - \mathbf{r}'|} \right\}, \tag{1}$$

where ϵ is the dielectric constant of the host materials (GaAs in this case) and $\mathbf{u}$ is the electron lattice displacement field. Notice that for the intralayer Coulomb interaction term we have used the linear continuous approximation for charge density fluctuations $\delta n/n_0 = -\nabla \cdot \mathbf{u}$ (this is correct in the absence of vacancies and interstitials). We choose to work in the symmetric gauge $\mathbf{A}(\mathbf{u}) = (-Bu_y/2\ ,\ Bu_x/2\ ,\ 0)$ where B is the applied magnetic field. In the absence of B the eigenmodes of the elastic medium can be labeled as either longitudinal or transverse; the magnetic field couples those two modes together and one has to diagonalize the Hamiltonian of the system anew. Nevertheless it is useful to decompose the displacement field $\mathbf{u}$ into transverse and longitudinal components u_T and u_L. After Fourier–transforming, the above Lagrangian becomes

$$L = n_0 \int \frac{d^2q}{(2\pi)^2} \left[\frac{1}{2} m \dot{u}_T^2 + \frac{1}{2} m \dot{u}_L^2 + \frac{1}{2} m\omega_c [\dot{u}_T u_L - \dot{u}_L u_T] - \frac{1}{2} m\omega_L^2 u_L^2 - \frac{1}{2} m\omega_T^2 u_T^2 \right], \tag{2}$$

where we have defined

$$\omega_L = \sqrt{c_L^2 q^2 + \frac{e^2 n_0}{2m\epsilon} q}, \quad \omega_T = c_T |\mathbf{q}|, \tag{3}$$

as the longitudinal and transverse zero magnetic field eigenfrequencies with

$$c_L = \sqrt{(\lambda + 2\mu)/mn_0}, \quad c_T = \sqrt{\mu/mn_0}, \tag{4}$$

and $\omega_c = eB/m$ is the cyclotron frequency. If we compare these modes with the results of Bonsall and Maradudin[15] for a WC with pure Coulomb interactions, we see that c_L is a higher order correction to the magnetoplasmon contribution (the second term on the right hand side of Eq. 3), and can in most cases be neglected.

We show in Appendix A that the new eigenfrequencies for the collective modes of the single layer electron system are given by

$$\omega_\pm^2 = \frac{1}{2}\left[\omega_c^2 + \omega_T^2 + \omega_L^2\right] \pm \frac{1}{2}\left[(\omega_c^2 + \omega_T^2 + \omega_L^2)^2 - 4\omega_T^2\omega_L^2\right]^{1/2}. \tag{5}$$

In the zero magnetic field limit ($\omega_c = 0$) the above modes decouple into pure longitudinal and transverse modes, while for $q \to 0$, the magnetoplasmon mode $\omega_+ \to \omega_c$, in accordance with Kohn's theorem.[12] For $\omega_c \gg \omega_L,\ \omega_T$, the eigenmode frequencies can be expanded as

$$\omega_+ = \omega_c + \frac{\omega_L^2 + \omega_T^2}{2\omega_c} + O(\omega_c^{-3}), \tag{6}$$

$$\omega_- = \frac{\omega_L \omega_T}{\omega_c} + O(\omega_c^{-3}). \tag{7}$$

The magnetophonon mode $\omega_- \sim q^{3/2}$ at long wavelengths, in agreement with Bonsall and Maradudin.[15]

III. BILAYER THEORY

We now consider a bilayer system composed of two parallel single layers A and B, separated by a distance d, with the magnetic field normal to the layers. In a continuum model, the interlayer Coulomb interactions include a long range term which accounts for interactions between density fluctuations in the two layers, and a short range term which accounts for the commensuration energy between the WCs in the two layers. The latter term imposes an energy penalty for out-of-phase fluctuations of the electron densities in the two layers, proportional to $(\mathbf{u}_A - \mathbf{u}_B)^2$. The Lagrangian for the bilayer system is then

$$\begin{aligned} L &= L_A + L_B + n_0 \int d^2r \Bigg[-\frac{1}{2}\frac{K}{n_0}(\mathbf{u}_A - \mathbf{u}_B)^2 \\ & -n_0 \int d^2r' \frac{e^2[\nabla \cdot \mathbf{u}_A(\mathbf{r})][\nabla' \cdot \mathbf{u}_B(\mathbf{r}')]}{4\pi\epsilon\sqrt{(x-x')^2+(y-y')^2+d^2}}\Bigg], \end{aligned} \tag{8}$$

where L_A, L_B are the individual single layer Lagrangians given by Eq. (2), and K is a "spring constant" for the out-of-phase fluctuations. When written in terms of the in-phase $\mathbf{v} = (\mathbf{u}_B + \mathbf{u}_A)/2$ and out-of-phase $\mathbf{u} = \mathbf{u}_B - \mathbf{u}_A$ field displacement modes, the Lagrangian decouples into independent in-phase and out-of-phase terms, so that there are two in-phase and two out-of-phase eigenmodes; the details are included in Appendix B. The in-phase modes correspond to a uniform translation of both layers, and are unimportant for the tunneling processes which are the topic of this paper. The dispersion of the out-of-phase modes is similar to Eq. (5) and given by

$$\begin{aligned} \Omega_\pm^2 &= \frac{1}{2}\left[\omega_c^2 + \Omega_T^2 + \Omega_L^2\right] \\ & \pm \frac{1}{2}\left[(\omega_c^2 + \Omega_T^2 + \Omega_L^2)^2 - 4\Omega_L^2\Omega_T^2\right]^{1/2}, \end{aligned} \tag{9}$$

where

$$\Omega_T^2 = c_T^2 q^2 + \frac{2K}{mn_0}, \tag{10}$$

$$\Omega_L^2 = c_L^2 q^2 + \frac{2K}{mn_0} + \frac{e^2 n_0}{2m\epsilon} q(1 - e^{-qd}). \tag{11}$$

Both of these modes are gapped at long wavelengths due to the short range interlayer coupling K. In the high magnetic field limit we obtain for the magnetoplasmon (Ω_+) and magnetophonon (Ω_-)

$$\Omega_+ = \omega_c + \frac{\Omega_L^2 + \Omega_T^2}{2\omega_c} + O(\omega_c^3), \tag{12}$$

$$\Omega_- = \frac{\Omega_L \Omega_T}{\omega_c} + O(\omega_c^3). \tag{13}$$

The fact that the magnetophonon mode is now gapped, contrary to the single layer case in which is gapless, is an important distinction between the single layer and bilayer case for the tunneling characteristics.

IV. TUNNELING CURRENT

So far we have developed an elastic theory for the electronic charge fluctuations in the bilayer system, whose low energy excitations can provide an energy dissipating mechanism necessary for the system to relax any charge defect created by tunneling events. What remains to be done is to couple these collective modes to the tunneling electrons in the system. Following JK we assume that the tunneling events are sufficiently infrequent that we may treat them as occurring independently, and hence develop a model of a single tunneling electron which is coupled to the WCs in each layer. Such a model can incorporate the effects of interlayer interactions (through the bilayer collective modes), but not interlayer coherence. The resulting coupling of a tunneling electron with the magnetophonons (the bilayer charge density fluctuations) is a standard electron–phonon coupling and is further discussed in Appendix C. The final form of the Hamiltonian is

$$H = H_0 + H_{\mathrm{bath}} + H_T^+ + H_T^-, \tag{14}$$

where

$$\begin{aligned} H_0 = &\left[\epsilon_A + i\sum_s M_{sA}\left(a_s^\dagger - a_s\right)\right] c_A^\dagger c_A \\ &+ \left[\epsilon_B + i\sum_s M_{sB}\left(a_s^\dagger - a_s\right)\right] c_B^\dagger c_B, \end{aligned} \tag{15}$$

$$H_{\mathrm{bath}} = \sum_s \hbar\Omega_s a_s^\dagger a_s, \tag{16}$$

$$H_T^+ = T c_A^\dagger c_B, \quad H_T^- = T c_B^\dagger c_A. \tag{17}$$

In the above $M_{sA(B)}$ are the electron–magnetophonon couplings and $\epsilon_{A(B)}$ the corresponding Madelung energies for the two Wigner crystal lattices, c and $c^\dagger$ are the tunneling electron annihilation and creation operators, a_s and $a_s^\dagger$ are the annihilation and creation operators for the collective modes, and T is the interlayer tunneling matrix element.[13] The collective modes that couple to a tunneling electron provide a mechanism to generate a "shake-up" that will relax the defect it has produced and dissipate the extra bias energy the tunneling electron has acquired.

The tunneling current related to the above Hamiltonian will be derived from Fermi's golden rule and has the following form[9]

$$\begin{aligned} I(V) &= \frac{e}{\hbar^2}\int_{-\infty}^{+\infty} dt e^{ieVt/\hbar}\langle\left[H_T^-(t), H_T^+(0)\right]\rangle \\ &= \frac{e}{\hbar^2}\int_{-\infty}^{+\infty} dt \left[e^{\frac{ieVt}{\hbar}} I^{\mp}(t) - e^{-\frac{ieVt}{\hbar}} I^{\pm}(t)\right] \end{aligned} \tag{18}$$

where the correlation function definitions are

$$\begin{aligned} I^{\mp}(t) &= \langle H_T^-(t) H_T^+(0)\rangle, \\ I^{\pm}(t) &= \langle H_T^+(t) H_T^-(0)\rangle, \end{aligned} \tag{19}$$

and the time–dependence is meant in the interaction picture representation, namely

$$H_T^{\pm}(t) = e^{\frac{i}{\hbar}H_0 t} H_T^{\pm} e^{-\frac{i}{\hbar}H_0 t}. \tag{20}$$

For the calculation of the correlation functions in Eq. (19) we use the same approach as JK. We assume the tunneling process is statistically independent from the collective mode propagation and can thus be averaged independently. For the statistical averaging of the collective modes we use the linked cluster expansion method[14] which turns out to involve only one link in the exponential resummation. In Appendix D we show in more detail how the calculation proceeds. For the correlation function we obtain

$$I^{\mp}(t) = \nu(1-\nu)T^2 C(t) \tag{21}$$

where

$$\begin{aligned} C(t) = \exp\Big\{ -\sum_s \frac{(M_{sB}-M_{sA})^2}{(\hbar\Omega_s)^2} \\ \times \Big[\big(N_s+1\big)\big(1-e^{-i\Omega_s t}\big) + N_s\big(1-e^{i\Omega_s t}\big) \Big] \Big\}, \end{aligned} \tag{22}$$

and N_s is the boson thermal occupation number for the collective modes of the system. To get the form of Eq. (19) it suffices to interchange A and B in Eq. (22). Notice that the in-phase modes drop out from the above correlation function due to their matrix element property $M_{sA} = M_{sB}$, this is not the case for the out-of-phase modes which, as we show in the Appendix, obey $M_{sA} = -M_{sB}$.

The experimental temperature range in the tunneling current results is of the order of 0.1 K$\sim 10^{-5}$ eV while the bias voltage is in the range of mV, so a zero temperature calculation is appropriate; this simplifies things considerably since the bosonic occupation numbers $N_s = 0$. In addition, due to the high magnetic field the magnetoplasmon modes will have a large gap and will not contribute to the electron coupling. Gathering all this together and switching to dimensionless units for the momentum integration ($x = q/q_0$) we find the following result for the time dependent correlation function

$$C(t) = \exp\Big[\int_0^1 dx f(x)\big(e^{-i\omega(x)t} - 1\big) \Big]. \tag{23}$$

In Appendix D we provide the analytic expression for the weight–function $f(x)$, while $\omega(x)$ is given by Eq. (13) or specifically by Eq. (D6). By differentiating Eq. (23) and taking the Fourier transform we obtain the following equation for the Fourier transform of the correlation function

$$\omega C(\omega) = \int_0^1 dx f(x)\omega(x) C(\omega - \omega(x)). \tag{24}$$

As we show in Appendix D this correlation function is zero for $\omega \le 0$.

The integral equation in Eq. (24) is difficult to solve analytically. However, we can find the asymptotic behavior of $C(\omega)$ by expanding in $\omega(x)$. To lowest order we obtain a first order differential equation with the solution

$$C(\omega) \sim \exp\Big[-\frac{(\omega - c_1)^2}{2c_2} \Big], \tag{25}$$

where we provide analytic expressions for c_1 and c_2 in Appendix D. This expansion gives us an analytic prediction for the peak bias voltage, namely c_1, and the general behavior of the correlation function at large bias voltages. We can investigate the behavior of c_1 in the two limiting cases when the interlayer Coulomb interaction is much weaker than the intralayer one and vice versa. We find that for the $d \gg a_0$ limit, c_1 scales as $1/a_0$, and the width of the Coulomb barrier peak $\sqrt{c_2}$ scales as $\sqrt{c_2} \sim 1/a_0^2\sqrt{B}$. This is the same phenomenological behavior JK extract from their calculations. In the opposite limit $d \ll a_0$, we find that $c_1 \sim d^2/a_0^3$ and $\sqrt{c_2} \sim d/a_0^3\sqrt{B}$. This limit is absent in the JK model. We should notice here that in this regime the actual behavior of the system will be significantly modified by coherence effects but those effects will be pronounced at zero-bias. At finite bias values we see from experimental results[3] that the Coulomb barrier peak survives, but it is “red-shifted” significantly which, as we see, our model is able to reproduce as a limiting behavior.

The importance of the asymptotic result lies on the insight it provides on the tunneling current behavior. We use this information to build an *Ansatz* for the correlation function. We are after a qualitative analytic expression that will capture the basic physics and reproduce the Gaussian asymptotic behavior. We choose the following *Ansatz*

$$C(\omega) = N\omega^r e^{-\lambda\omega^2}. \tag{26}$$

As we show in Appendix E, this choice reproduces very accurately the numerical solution of the equation which is our strongest justification for using it. For the parameters involved in this choice of $C(\omega)$ we develop a self consistent method of evaluation. This method is based on the moment expansion of $C(\omega)$. If we multiply–differentiate Eq. (23) we end up to the following moment equations

$$\int_0^\infty d\omega C(\omega) = 2\pi, \tag{27}$$

$$\int_0^\infty d\omega \omega C(\omega) = 2\pi c_1, \tag{28}$$

$$\int_0^\infty d\omega \omega^2 C(\omega) = 2\pi\big(c_2 + c_1^2\big). \tag{29}$$

These are the three equations that our Ansatz should obey self–consistently. We show in Appendix E in more detail how we extract the values for N, r and λ from them. Our theoretical value for the peak bias voltage

can be easily calculated now from Eq. (26). The result is

$$V_0 = \frac{1000\hbar}{e}\sqrt{\frac{r}{2\lambda}} \tag{30}$$

where we have converted it to mV. In Fig. (1) we present the results the moment expansion method produces. The theoretical parameters are directly taken from the experiment.[7] The bilayer sample area is S=0.0625mm^2 and the single layer electron density is n_0=1.6 $\times$ 10^{11} cm^{-2}. The perpendicular magnetic field varies from 8 T to 13.75 T and the Wigner crystal lattice parameter has the value $a_0 \simeq 270$ Å which corresponds to the stable hexagonal lattice configuration ($n_0 = 2/\sqrt{3}a_0^2$).[15] The double well separation distance is d=175 Å and the dielectric constant of GaAs is ϵ=12.9ϵ_0 $\simeq$1.14 $\times$ 10^{-11} F/m. The transverse sound velocity for the electron gas is given by[15]

$$c_T \simeq \sqrt{0.0363\frac{e^2}{\sqrt{3}\epsilon m a_0}} \simeq 53552 \text{ m/s}. \tag{31}$$

For the electron mass we use the electron effective mass value in the GaAs background m=0.067m_e. For the K parameter we notice that by construction K/n_0 is a measure of the energy density per electron due to the short range part of the interlayer Coulomb interactions. So we assume it will scale accordingly as

$$\frac{K}{n_0} = \kappa\frac{e^2/4\pi\epsilon d}{\pi l^2},\ \kappa > 0. \tag{32}$$

The value of κ is a measure of the magnetophonon gap in this system. Côté and collaborators have performed time–dependent Hartree Fock calculations investigating the magnetophonon dispersion relation and crystalline phases for the bilayer system.[16] They were able to provide us with a κ=0.0085 value. With this last piece of the puzzle in place we are able to produce our analytic results for the tunneling current shown in Fig. (1). As we see, the model captures the qualitative behavior of the current and it accurately reproduces the experimental results for the peak bias voltages associated with different applied magnetic field values. Notice though, that the pronounced experimental low bias suppression[7] is lifted in our case although present in the JK model[9] and in spectral weight calculations[17] associated with this bilayer system. We attribute this behavior to the very small value of κ and mostly to the role of interlayer interactions. In particular, as it can be seen in Fig. (1), the Coulomb blockade tends to be restored with increasing magnetic field because the role of interlayer interactions is diminished in that limit. We observe the same behavior as well when the interlayer distance d is increased, which signifies the same physical limit, where essentially the JK results are recovered. We believe that in order to capture all quantitative characteristics of the tunneling current for the whole range of bias voltages one has to go beyond the continuum approximation employed here.

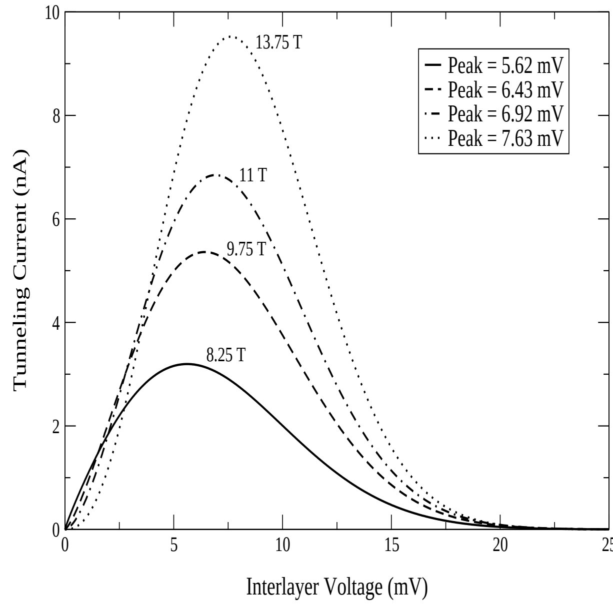

FIG. 1: Tunneling current curves for different magnetic field values using the moment expansion solution of Eq. (24). The legend shows the peak bias values calculated by Eq. (30).

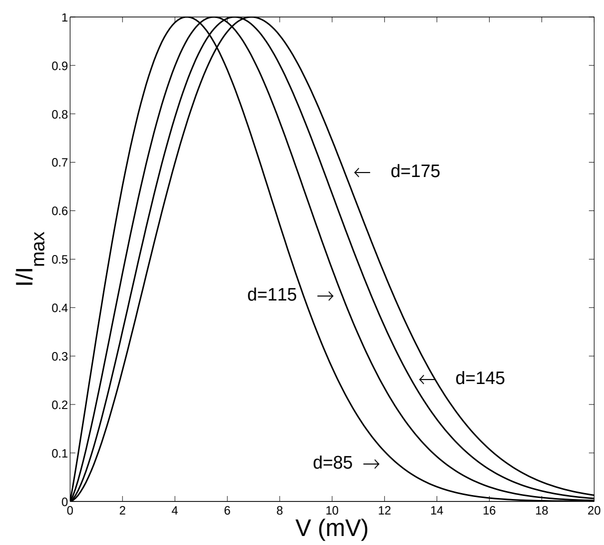

FIG. 2: Normalized tunneling current curves for different interlayer separation distances d measured in Å.

Another way of calculating the correlation function is by numerically integrating Eq. (24). In Appendix E we report analytically on this approach. The results we get are shown in Fig. (3) and are very similar to the moment expansion results; the same applies for the peak bias values.

At this point we are in position to analytically investigate how our solution behaves for different interlayer separation distances d. Notice that the dependence in d is introduced in two places in our model, the one is the tunneling matrix elements T (which depends exponen-

tially on d) and the other is through the long-range part of the interlayer Coulomb interaction. Since we are interested in the latter only, we will normalize the I–V curves for different d values to the peak current value (this has the effect of removing the tunneling matrix element contribution). According to experimental results[8] we expect to see a "red-shift" in the I–V curves associated with the bounding interlayer exciton energy $-e^2/\epsilon d$. Our results are shown in Fig. (2) and as we see our model captures this kind of important interlayer correlation physics. According to our model, we can conclude that the effect of the interlayer Coulomb interaction in the real system is two-fold. First, the short-range correlations derived from the interlayer interaction introduce a gap in the low frequency excitations, which is well pronounced in the experimental results,[7] and second, the long-range effects of the interlayer interactions "soften" the response of the bilayer system to tunneling, through the exciton creation associated with tunneling events.

V. SUMMARY

We have introduced a model for tunneling in a bilayer system in high magnetic fields, which explicitly includes the interlayer interactions. The bilayer system of electrons is modeled as two correlated Wigner crystals in the continuum limit. The low energy collective modes of the crystals are coupled to the tunneling electrons in the system, providing an energy dissipation mechanism to relax the charge defect produced by the tunneling event. This model allows for an approximate analytic solution to the tunneling current problem that captures the basic experimental behavior of the tunneling current curve and fairly accurately reproduces the peak bias values for different applied magnetic field cases along with the expected I–V dependence with interlayer distance d. In the future we would like to find a way to include coherence into the model and reproduce the zero bias peak anomaly[3] the tunneling current exhibits.

VI. ACKNOWLEDGMENTS

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APPENDIX A: SINGLE LAYER EIGENMODES

We have expressed the single layer dynamics by writing down the Lagrangian of Eq. (2). The canonical quantization process is a textbook exercise where one has to solve the Lagrange equation of motion for the fields, namely

$$\ddot{u}_T + \omega_c \dot{u}_L + \omega_T^2 u_T = 0, \tag{A1}$$

$$\ddot{u}_L - \omega_c \dot{u}_T + \omega_L^2 u_L = 0. \tag{A2}$$

Notice the effect that the magnetic field has in coupling the otherwise orthogonal longitudinal and transverse zero field eigenmodes. The above system of equations is easy to prove that has eigenvalues given by Eq. (5). The general solution for the eigenmodes is of the form

$$u_T = A_T e^{i\omega_+ t} + A'_T e^{-i\omega_+ t} + B_T e^{i\omega_- t} + B'_T e^{-i\omega_- t}, \tag{A3}$$

$$\begin{aligned} u_L = {} & i\frac{\omega_c\omega_+}{\omega_+^2 - \omega_L^2}[-A_T e^{i\omega_+ t} + A'_T e^{-i\omega_+ t}] \\ & + i\frac{\omega_c\omega_-}{\omega_-^2 - \omega_L^2}[-B_T e^{i\omega_- t} + B'_T e^{-i\omega_- t}], \end{aligned} \tag{A4}$$

where A_T, A'_T and B_T, B'_T are the coefficients associated with the creation and annihilation of the two modes respectively. These are directly related to the creation and annihilation operators by the normalization condition. The way we reach to the latter is by expressing all the higher derivative terms in terms of the canonical displacements and the canonical momenta at time equal to zero which obey the commutation relations $[u_{i_0}(\mathbf{q}), p_{j_0}(\mathbf{q}')] = i\hbar\delta_{ij}(2\pi)^2\delta^2(\mathbf{q}-\mathbf{q}')$. After a fair amount of algebra we have convinced ourselves that the operators obey the correct eigenmode commutation relation. By applying the normalization condition we can reach the final form of the eigenmode operators given by (implicit $\mathbf{q}$ dependence will be suppressed unless necessary)

$$\begin{aligned} a_1(\mathbf{q}) \; = {} & \tilde{a}_1\Big[\frac{\omega_c\omega_+}{n_0 m}p_{L_0} - i\omega_c\frac{\omega_+^2+\omega_L^2}{2}u_{L_0} + i\frac{\omega_+^2-\omega_L^2}{n_0 m}p_{T_0} \\ & + \omega_+\Big(\omega_+^2 - \omega_L^2 - \frac{\omega_c^2}{2}\Big)u_{T_0}\Big], \end{aligned} \tag{A5}$$

$$\begin{aligned} a_2(\mathbf{q}) \; = {} & \tilde{a}_2\Big[-\frac{\omega_c\omega_-}{n_0 m}p_{L_0} + i\omega_c\frac{\omega_-^2+\omega_L^2}{2}u_{L_0} + i\frac{\omega_L^2-\omega_-^2}{n_0 m}p_{T_0} \\ & + \omega_-\Big(\omega_L^2 - \omega_-^2 + \frac{\omega_c^2}{2}\Big)u_{T_0}\Big], \end{aligned} \tag{A6}$$

where they obey $[a_i(\mathbf{q}), a_j^\dagger(\mathbf{q}')] = (2\pi)^2\delta^2(\mathbf{q}-\mathbf{q}')\delta_{ij}$ and

$$\tilde{a}_1 = \sqrt{\frac{mn_0}{2\hbar\omega_+(\omega_+^2-\omega_L^2)(\omega_+^2-\omega_-^2)}}, \tag{A7}$$

$$\tilde{a}_2 = \sqrt{\frac{mn_0}{2\hbar\omega_-(\omega_L^2-\omega_-^2)(\omega_+^2-\omega_-^2)}}. \tag{A8}$$

We can finally express the displacement fields in terms of the eigenmodes in Schrödinger's representation as

$$\begin{aligned} u_T(\mathbf{q}) \; = {} & \sqrt{\frac{\hbar}{2mn_0(\omega_+^2-\omega_-^2)}}\Big[\sqrt{\frac{\omega_+^2-\omega_L^2}{\omega_+}}(a_1^\dagger + a_1) \\ & + \sqrt{\frac{\omega_L^2-\omega_-^2}{\omega_-}}(a_2^\dagger + a_2)\Big], \end{aligned} \tag{A9}$$

$$u_L(\mathbf{q}) = i\sqrt{\frac{\hbar\omega_c^2}{2mn_0(\omega_+^2-\omega_-^2)}}\Bigg[\sqrt{\frac{\omega_+}{\omega_+^2-\omega_L^2}}(a_1-a_1^\dagger) - \sqrt{\frac{\omega_-}{\omega_L^2-\omega_-^2}}(a_2-a_2^\dagger)\Bigg], \quad \text{(A10)}$$

APPENDIX B: BILAYER EIGENMODES

Starting with Eq. (8) and introducing the in-phase $\mathbf{v} = (\mathbf{u}_B + \mathbf{u}_A)/2$ and out-of-phase $\mathbf{u} = \mathbf{u}_B - \mathbf{u}_A$ modes, we find that the Lagrangian can be written as $L = L_{\text{in}} + L_{\text{out}}$, with

$$L_{\text{in}} = n_0\int\frac{d^2q}{(2\pi)^2}\frac{1}{2}(2m)\Bigg[\dot v_L^2 + \dot v_T^2 + \omega_c(\dot v_T v_L - \dot v_L v_T) - \omega_T^2 v_T^2 - O_L^2 v_L^2\Bigg], \quad \text{(B1)}$$

$$L_{\text{out}} = n_0\int\frac{d^2q}{(2\pi)^2}\frac{1}{2}\frac{m}{2}\Bigg[\dot u_L^2 + \dot u_T^2 + \omega_c(\dot u_T u_L - \dot u_L u_T) - \Omega_T^2 u_T^2 - \Omega_L^2 u_L^2\Bigg], \quad \text{(B2)}$$

where

$$\Omega_T^2 = c_T^2 q^2 + \frac{2K}{mn_0}, \quad \text{(B3)}$$

$$\Omega_L^2 = c_L^2 q^2 + \frac{2K}{mn_0} + \frac{e^2 n_0}{2m\epsilon}q(1-e^{-qd}), \quad \text{(B4)}$$

$$O_L^2 = c_L^2 q^2 + \frac{e^2 n_0}{2m\epsilon}q(1+e^{-qd}), \quad \text{(B5)}$$

$$O_T = \omega_T. \quad \text{(B6)}$$

We see that the four modes have decoupled into two in-phase (with total mass $2m$) and two out-of-phase (with reduced mass $m/2$) modes that are governed by effective single layer Lagrangian dynamics. Notice that the in-phase transverse mode remained unchanged. In order to obtain the analytic results for the out-of-phase modes displacement field operators we have to redefine the parameters in Eqs. (A5-A10) as $m \to m/2$, $\omega_T^2 \to \Omega_T^2$, $\omega_L^2 \to \Omega_L^2$ while for the in-phase mode we have $m \to 2m$, $\omega_L^2 \to O_L^2$. The new eigenvalues $\Omega_\pm$ will be given by Eq. (9) and a corresponding one from Eq. (5) for $O_\pm$ when we carry out the above frequency changes. Notice that the mass in the cyclotron frequency formula does not change in either case. We will use the operator form of these eigenmodes to calculate below their coupling matrix elements with the electrons in the system.

APPENDIX C: COLLECTIVE MODE COUPLING MATRIX

The coupling between a point charge with charge density $n_e(\mathbf{r})$ and a charge fluctuation $\delta n(\mathbf{r})$ in either of the two layers is given by

$$H_{\text{cpl}} = \frac{e^2}{4\pi\epsilon}\int d^2r\int d^2r_A\frac{n_e(\mathbf{r})\delta n_A(\mathbf{r}_A)}{|\mathbf{r}-\mathbf{r}_A|} + \frac{e^2}{4\pi\epsilon}\int d^2r\int d^2r_B\frac{n_e(\mathbf{r})\delta n_B(\mathbf{r}_B)}{|\mathbf{r}-\mathbf{r}_B-\mathbf{d}|}. \quad \text{(C1)}$$

In the continuum approximation to lowest order we have $\delta n_A = -n_0\nabla\cdot\mathbf{u}_A$, and we can place the electron at the origin so that $n_e(\mathbf{r}) = \delta^{(2)}(\mathbf{r})$. After introducing the in-phase and out-of-phase displacement fields and Fourier transforming, the coupling becomes

$$H_{\text{cpl}} = (c_A^\dagger c_A - c_B^\dagger c_B)\int\frac{d^2q}{(2\pi)^2}\frac{e^2n_0}{4\epsilon q}(1-e^{-qd})i\mathbf{q}\cdot\mathbf{u}, \quad \text{(C2)}$$

where we have dropped a constant term (which depends upon v). Next, we use the operator definition for the out-of-phase displacement field found previously

$$u_L = -if_1a_1^\dagger + if_1a_1 - if_2a_2^\dagger + if_2a_2, \quad \text{(C3)}$$

where

$$f_1 = \omega_c\sqrt{\frac{\hbar}{n_0m}\frac{\Omega_+}{(\Omega_+^2-\Omega_L^2)(\Omega_+^2-\Omega_-^2)}}, \quad \text{(C4)}$$

$$f_2 = -\omega_c\sqrt{\frac{\hbar}{n_0m}\frac{\Omega_-}{(\Omega_L^2-\Omega_-^2)(\Omega_+^2-\Omega_-^2)}}, \quad \text{(C5)}$$

and the coupling term assumes the form of Eq. (15), where the matrix elements are given by

$$M_{sA} = -M_{sB} = -\frac{e^2n_0}{4\epsilon}(1-e^{-dq})f_s, \quad \text{(C6)}$$

and $s = 1, 2$ refers to the two out-of-phase bilayer eigenmodes but any summation on it is meant to include an integration in q as well.

APPENDIX D: CORRELATION FUNCTION

To calculate the correlation function given by Eq. (19) we use the linked cluster expansion method.[14] As it turns out there is only one link involved in the exponential resummation of the series. As pointed out by JK, if we assume that the tunneling events are faster than the relaxation time for WCs, then the thermal averaging of the magnetomodes and the tunneling electrons is statistically independent. The correlation function becomes

$$I^{\mp}(t) = T^2\nu(1-\nu)\langle T_t\exp\Bigg[-\frac{i}{\hbar}\int_0^t dt' e^{\frac{iH_it'}{\hbar}} \times (H_f - H_i)e^{-\frac{iH_it'}{\hbar}}\Bigg]\rangle \quad \text{(D1)}$$

where

$$H_i = \epsilon_A - \Delta_A + \sum_s \hbar\Omega_s \big(a_s^\dagger - \frac{iM_{sA}}{\hbar\Omega_s}\big)\big(a_s + \frac{iM_{sA}}{\hbar\Omega_s}\big), \quad \text{(D2)}$$

$$H_f = \epsilon_B - \Delta_B + \sum_s \hbar\Omega_s \big(a_s^\dagger - \frac{iM_{sB}}{\hbar\Omega_s}\big)\big(a_s + \frac{iM_{sB}}{\hbar\Omega_s}\big), \quad \text{(D3)}$$

are the Hamiltonians of the system before and after a tunneling event respectively. Using the linked cluster expansion method we obtain the result of Eq. (21), aside from an overall phase factor $(\epsilon_B - \Delta_B - \epsilon_A + \Delta_A)t/\hbar$ which we assume to be zero. As JK point out, this amounts to a complete Wigner crystal relaxation after each tunneling event, where the polaron shifts $\Delta_A - \Delta_B$ defined by

$$\Delta_{A(B)} = \sum_\alpha \frac{|M_{\alpha A(B)}|^2}{\hbar\omega_\alpha}, \quad \text{(D4)}$$

compensate the Madelung energies $\epsilon_B - \epsilon_A$ of the interstitial positions. Another point of interest is that as we see from Eq. (22) the matrix elements of the electron coupling with the collective modes of the system enter in the correlation function formula as a difference, so that the tunneling electron only couples to the out-of-phase modes. When an electron tunnels from one layer to the other the remaining electrons in that layer have to fill in the hole; at the same time the electrons in the receiving layer need to "open up" and create a hole for the incoming electron, so we see that the relative motion of the two charge densities should be out-of-phase during a tunneling event.

Since the magnetoplasmons have a large gap in high fields we only need to consider the magnetophonon matrix element, which in the high magnetic field case will be given by

$$f(x) = \frac{c\omega_c^5}{c_T^8 q_0^6} \frac{x(1-e^{-\gamma x})^2}{\delta + x^2 + 2\alpha + \beta x(1-e^{-\gamma x})} \frac{1}{\sqrt{x^2+\alpha}} \times \frac{1}{[\alpha + \beta x(1-e^{-\gamma x})]^{3/2}} \frac{1}{\delta - \alpha - x^2}. \quad \text{(D5)}$$

For the magnetophonon frequency we will have

$$\omega(x) = \frac{c_T^2 q_0^2}{\omega_c} \sqrt{\alpha + \beta x(1-e^{-\gamma x})}\sqrt{x^2+\alpha}. \quad \text{(D6)}$$

For convenience we have introduced dimensionless units for the momenta ($x = q/q_0$), and we have set $c_L = 0$. The parameters in the equations above are

$$c = \frac{n_0}{8\pi\hbar m}\left(\frac{e^2}{\epsilon}\right)^2, \quad \text{(D7)}$$

$$\alpha = \frac{2K}{mn_0 c_T^2 q_0^2} = \frac{\kappa}{2\pi^2 m c_T^2} \frac{1}{(lq_0)^2} \frac{e^2}{\epsilon d}, \quad \text{(D8)}$$

$$\beta = \frac{e^2 n_0}{2\epsilon m c_T^2 q_0}, \quad \text{(D9)}$$

$$\gamma = dq_0, \quad \text{(D10)}$$

$$\delta = \left(\frac{\omega_c}{c_T q_0}\right)^2. \quad \text{(D11)}$$

The parameter α is dimensionless and gives a measure of the magnetophonon gap. We have used Eq. (32) to produce the final equation that defines it. The parameter γ is dimensionless and measures the relative interaction strength between the interlayer and intralayer Coulomb interactions. The parameter β is dimensionless as well and does not depend on the Wigner crystal lattice parameter a_0 as it turns out if the dependence of c_T given by Eq. (31) is taken into account. In order to derive the result for the asymptotic behavior of the correlation function we have to Taylor–expand $C(\omega)$ involved in the integral of Eq. (24) in powers of $\omega(x)$. It is straightforward to show that to lowest order the first order differential equation we obtain has the Gaussian solution given by Eq. (25). The coefficients involved in the expansion are given by

$$\begin{aligned} c_1 &= \int_0^1 dx f(x)\omega(x) \\ &= c\frac{\omega_c^4}{c_T^6 q_0^4} \int_0^1 dx \frac{x(1-e^{-\gamma x})^2}{\delta + x^2 + 2\alpha + \beta x(1-e^{-\gamma x})} \\ &\times \frac{1}{\alpha + \beta x(1-e^{-\gamma x})} \frac{1}{\delta - \alpha - x^2}, \end{aligned} \quad \text{(D12)}$$

$$\begin{aligned} c_2 &= \int_0^1 dx f(x)\omega^2(x) \\ &= c\frac{\omega_c^3}{c_T^4 q_0^2} \int_0^1 dx \frac{x(1-e^{-\gamma x})^2}{\delta + x^2 + 2\alpha + \beta x(1-e^{-\gamma x})} \\ &\times \sqrt{\frac{x^2+\alpha}{\alpha + \beta x(1-e^{-\gamma x})}} \frac{1}{\delta - \alpha - x^2}. \end{aligned} \quad \text{(D13)}$$

As we mentioned in the text we can expand to one order higher and analytically solve the differential equation as well, but the correction will not be very useful since these results are valid only in the asymptotic region that a lowest order calculation suffices to catch the essential characteristics. In the $d \gg a_0$ case we can ignore the exponentials in the integrand of Eqs. (D12, D13) and as it turns out a $c_1 \sim 1/a_0$ and $\sqrt{c_2} \sim 1/a_0^2 B$ scaling behavior emerges. In the opposite limit $d \ll a_0$, if we expand the exponentials in the integrand of Eqs. (D12, D13) we find that $c_1 \sim d^2/a_0^3$ and $\sqrt{c_2} \sim d/a_0^3 B$.

At this point we would like to justify the assumption that the correlation function is zero for zero or negative bias voltage ($\omega \leq 0$). By expanding the exponential in Eq. (24) and interchanging summation with integration we can perform the last integral that gives a delta function in the frequencies. Since the magnetophonons are always positive definite (gapped) we see that the correlation function

$$\begin{aligned} C(\omega) =& 2\pi e^{-\int_0^1 dx f(x)} \sum_{n=0}^\infty \frac{1}{n!} \int_0^1 dx_1 f(x_1) \cdots \int_0^1 dx_n f(x_n) \\ &\times \delta\big(\omega - \omega(x_1) - \cdots - \omega(x_n)\big) \end{aligned} \quad \text{(D14)}$$

has to be zero for any ω less than the lowest possible magnetophonon frequency value.

With this last piece of information about the correlation function we can proceed now into developing our method of moment expansion. As we have already justified we use the Ansatz of Eq. (26) and the three different moments of the correlation function given by Eqs. (27–29). At this point it is convenient to make a change of variable $\omega = eV/1000\hbar$ to convert the argument of $C(\omega)$ into mV and define the corresponding parameter in the exponent as $\Lambda = \lambda\left(\frac{e}{1000\hbar}\right)^2$. By performing the integrals the three moment equations become

$$\frac{1}{2}N\left(\frac{e}{1000\hbar}\right)^{r+1}\Lambda^{-\frac{r+1}{2}}\Gamma\left(\frac{r+1}{2}\right) = 2\pi, \tag{D15}$$

$$\frac{1}{2}N\left(\frac{e}{1000\hbar}\right)^{r+2}\Lambda^{-\frac{r+2}{2}}\Gamma\left(\frac{r+2}{2}\right) = 2\pi c_1, \tag{D16}$$

$$\frac{1}{2}N\left(\frac{e}{1000\hbar}\right)^{r+3}\Lambda^{-\frac{r+3}{2}}\Gamma\left(\frac{r+3}{2}\right) = 2\pi\left(c_2 + c_1^2\right). \tag{D17}$$

We can divide Eq. (D16) and Eq. (D17) by Eq. (D15) and then equating the two we reach the self consistent equation for r, namely

$$\frac{\Gamma^2\left(\frac{r+2}{2}\right)}{\Gamma\left(\frac{r+3}{2}\right)} = \frac{\Gamma\left(\frac{r+1}{2}\right)}{1 + c_2/c_1^2}. \tag{D18}$$

We numerically solve the above equation and obtain a value for r. The usual range of r for the magnetic field values considered is $1/2 < r < 2$. This can be regarded as an estimate for the low-bias current power-low behavior. Having r at hand we can go back and evaluate the rest of the parameters. The final form of the correlation function becomes

$$C(V) = N\left(\frac{e}{1000\hbar}\right)^{r} V^r e^{-\Lambda V^2} \tag{D19}$$

where V measures in mV. To obtain the peak bias value result of Eq. (30) we find the root of the first derivative of the above equation.

APPENDIX E: NUMERICAL INTEGRATION

We have numerically integrated the integral equation for the correlation function in two different ways—first by a direct integration of the integral equation and then by introducing the density of states (similar to the JK method). As both methods give the same results we will only report on the latter. The equation to be integrated is

$$zC(z) = \int_0^1 dx f(x)\frac{\omega(x)}{\gamma_0}C\left(z - \frac{\omega(x)}{\gamma_0}\right)\theta\left(z - \frac{\omega(x)}{\gamma_0}\right) \tag{E1}$$

where we have introduced the parameter $\gamma_0 = \frac{e}{1000\hbar}$ to convert the frequency argument of the correlation function into mV. Before we proceed we should notice that the magnetophonon frequency is bounded in a region $\alpha_1 \le \frac{\omega(x)}{\gamma_0} \le \alpha_2$ which means that the density of states is non zero only in that range. The values of α_1 and α_2 are given by substituting $x = 0$ and $x = 1$ into Eq. (D6) respectively. The upper bound α_2 appears due to the momentum cutoff we have introduced. The final form for the correlation function integral equation will be

$$C(z) = \begin{cases} \frac{1}{z}\int_{\alpha_1}^{z} dy g(y) C(z-y), & \alpha_1 \le z \le \alpha_2, \\ \frac{1}{z}\int_{\alpha_1}^{\alpha_2} dy g(y) C(z-y), & z \ge \alpha_2, \end{cases} \tag{E2}$$

where the definition for the density of states is

$$g(y) = y\left(\frac{f(x)}{\frac{1}{\gamma_0}\left|\frac{d\omega(x)}{dx}\right|}\right)_{x(y)}, \tag{E3}$$

and $x(y)$ is the root of the equation $\omega(x) = \gamma_0 y$. In this approach one has to "jump–start" the algorithm with an assumption for the low bias points. We can show that for small frequencies (but greater than the lower magnetophonon bound) the correlation function will have a $z - \alpha_1$ behavior. In Fig. (3) we show our results for the tunneling current using this method. As we see the qualitative behavior of the tunneling current is very similar to our analytic result, and the peak bias values are similar as well.

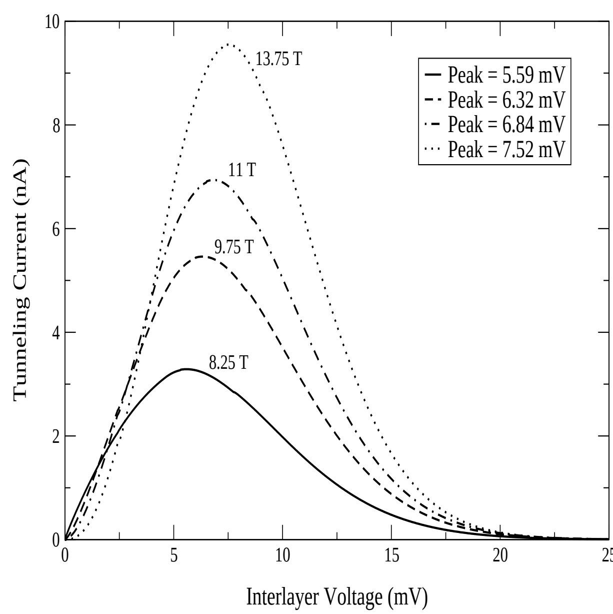

FIG. 3: Tunneling current curves for different magnetic field values produced by numerically integrating Eq. (24). The legend shows the peak bias values obtained with this approach. They are in close agreement with the analytic results.

[1] Y. W. Suen, L. W. Engel, M. B. Santos, M. Shayegan, and D. C. Tsui, Phys. Rev. Lett. **68**, 1379 (1992); J. P. Eisenstein, G. S. Boebinger, L. N. Pfeiffer, K. W. West, and Song He, Phys. Rev. Lett. **68**, 1383 (1992).

[2] S. Q. Murphy, J. P. Eisenstein, G. S. Boebinger, L. N. Pfeiffer, and K. W. West,Phys. Rev. Lett. **72**, 728 (1994).

[3] I. B. Spielman, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **84**, 5808 (2000).

[4] I. B. Spielman, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **87**, 036803 (2001).

[5] M. Kellogg, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West Phys. Rev. Lett. **93**, 036801 (2004).

[6] E. Tutuc, M. Shayegan, and D. A. Huse Phys. Rev. Lett. **93**, 036802 (2004).

[7] J. P. Eisenstein, L. N. Pfeiffer, and K. W. West Phys. Rev. Lett. **69**, 3804 (1992).

[8] J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **74**, 1419 (1995)

[9] P. Johansson and J. M. Kinaret, Phys. Rev. B **50**, 4671 (1994).

[10] A. J. Leggett, S. Chakravarty, A. T. Dorsey, A. Garg, M. P. A. Fisher, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987).

[11] See for example §4 in L. D. Landau and E. M. Lifshitz, *Theory of Elasticity* 3rd Edition

[12] W. Kohn, Phys. Rev. **123**, 1242 (1961)

[13] J. Bardeen, Phys. Rev. Lett. **6**, 57 (1961)

[14] See section 4.3.6 in G. D. Mahan, *Many-Particle Physics*, Third edition. Notice formula (4.369) needs an additional term $\frac{2t}{i\omega}$ to be complete.

[15] L. Bonsall and A. A. Maradudin, Phys. Rev. B **15**, 1959 (1977); H. Fukuyama, Solid State Commun. **17**, 1323 (1975)

[16] R. Côté *et al.* private communication.

[17] Song He, P. M. Platzman, and B. I. Halperin, Phys. Rev. Lett. **71**, 777 (1993).